

# 1. kolokvij iz *Opće teorije sustava* (grupa A)

**Zadatak 1.** Sustav opisan diferencijalnom jednažbom

$$4x^{(5)} + 18x^{(4)} + 32x^{(3)} + 28\ddot{x} + 12\dot{x} + 2x = 3u^{(5)} + 3u^{(3)} + \ddot{u} + 5u$$

prevedi u prostor stanja. Odredi amplitudu step pobude da stacionarno stanje izlazne varijable bude 1.

**Zadatak 2.** Nađi rješenje  $x_1(t)$ ,  $x_2(t)$  sustava

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\frac{1}{2}x_1 - x_2,\end{aligned}$$

za opće početne uvjete.

Odredi početne uvjete za koje će vrijediti: a)  $x_1(\pi) = 0$ ; b)  $x_2(\pi) = 0$ .

**Zadatak 3.** Za koje kombinacije vrijednosti parametara  $a$  i  $b$  sustav

$$\begin{aligned}\dot{x}_1 &= -2x_1 - x_3 + au_1 + bu_2, \\ \dot{x}_2 &= 2x_1 - 4x_2 - x_3 + 3u_1 + 4u_2, \\ \dot{x}_3 &= -x_1 - 2x_3 + 2u_1 + 5u_2, \\ y_1 &= ax_1 + 2x_2 + x_3, \\ y_2 &= bx_1 + 3x_2 + 5x_3,\end{aligned}$$

nije: a) upravljiv; b) mjerljiv.

**Napomena:** Konačna rješenja zadataka treba napisati na poleđini !!!  $\Rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow \Rightarrow$

Ime, prezime i smjer:

## Rješenja:

1.

Transfer function:

$$3s^5 + 3s^3 + s^2 + 5$$

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$$4s^5 + 18s^4 + 32s^3 + 28s^2 + 12s + 2$$

A =

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -1/2 & -3 & -7 & -8 & -9/2 \end{bmatrix}$$

B =

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1/4 \end{bmatrix}$$

$$C = [7/2, -9, -20, -21, -27/2]$$

$$D = 3/4$$

2.

$$\gg \text{expm}([0 \ 1; -0.5 \ -1]*t)$$

ans =

$$\begin{bmatrix} \exp(-1/2*t)*\cos(1/2*t)+\exp(-1/2*t)*\sin(1/2*t), & 2*\exp(-1/2*t)*\sin(1/2*t) \\ -\exp(-1/2*t)*\sin(1/2*t), & \exp(-1/2*t)*\cos(1/2*t)-\exp(-1/2*t)*\sin(1/2*t) \end{bmatrix}$$

3.

A =

$$\begin{bmatrix} -2 & 0 & -1 \\ 2 & -4 & -1 \\ -1 & 0 & -2 \end{bmatrix}$$

B =

$$\begin{bmatrix} a, b \\ 3, 4 \\ 2, 5 \end{bmatrix}$$

C =

$$\begin{bmatrix} a, 2, 1 \\ b, 3, 5 \end{bmatrix}$$

$$\begin{aligned}
\text{ans} &= \\
&2*u11-u31 \\
&2*u11-u31 \\
&-u11+2*u31 \\
\text{ans} &= \\
&-u12-u32 \\
&2*u12-3*u22-u32 \\
&-u12-u32 \\
\text{ans} &= \\
&u13-u33 \\
&2*u13-u23-u33 \\
&-u13+u33
\end{aligned}$$

$$\begin{aligned}
P &= \\
&[ \quad 0, \sqrt{1/3}, \sqrt{1/3}] \\
&[ \quad 1, \sqrt{1/3}, \sqrt{1/3}] \\
&[ \quad 0, -\sqrt{1/3}, \sqrt{1/3}]
\end{aligned}$$

$$\begin{aligned}
\text{inv}P &= \\
&[ \quad -1, \quad 1, \quad 0] \\
&[ \sqrt{3/4}, \quad 0, -\sqrt{3/4}] \\
&[ \sqrt{3/4}, \quad 0, \sqrt{3/4}]
\end{aligned}$$

$$\begin{aligned}
B_m &= \\
&[ \quad \quad -a+3, \quad \quad -b+4] \\
&[ \quad 1/2*3^{1/2}*a-3^{1/2}, \quad 1/2*3^{1/2}*b-5/2*3^{1/2}] \\
&[ \quad 1/2*3^{1/2}*a+3^{1/2}, \quad 1/2*3^{1/2}*b+5/2*3^{1/2}]
\end{aligned}$$

$$\begin{aligned}
C_m &= \\
&[ \quad \quad 2, \quad 1/3*3^{1/2}*a+1/3*3^{1/2}, \quad 1/3*3^{1/2}*a+3^{1/2}] \\
&[ \quad \quad 3, \quad 1/3*3^{1/2}*b-2/3*3^{1/2}, \quad 1/3*3^{1/2}*b+8/3*3^{1/2}]
\end{aligned}$$